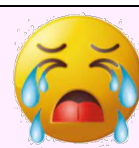
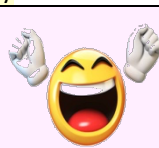


Analysis and Approaches Topic Checklist

Standard and Higher

Higher only

IB - A & A



Number and Algebra

Standard Form				
Arithmetic Series				
Geometric series (including sum of infinite geometric series)				
Sigma Notation				
Series applications (simple Interest, compound Interest, depreciation/population growth/spread of disease)				
Indices rules – multiplication, division and negative powers				
Logs – Index rule and natural logarithms				
Approximation, decimal places, significant figures				
Upper and lower bounds of rounded numbers.				
Percentage errors				
Estimation				
Amortization and annuities using technology				
Use technology to solve: <ul style="list-style-type: none"> Systems of linear equations in up to 3 variables Polynomial equations 				
Solving logs and exponential equations (including hidden quadratics with exponentials)				
Logs – 5 rules (index, power, multiplication, division, change of base)				
Indices Rules – rational powers and getting like bases in order to simplify				
The sum of infinite geometric sequences				
Complex numbers				
Matrices				
Eigenvalues and eigenvectors				

Functions

3 forms of a straight line (gradient intercept, general, point-gradient)				
Gradients and intercepts				
Midpoint and distances				
Straight Line Graphs – finding equations				
Parallel lines				
Perpendicular lines				
Functions – basic concept, notation and domain and range				
Functions - inverse (inverse function reverses or undoes the effect of a function). Concept of inverse function as a reflection in the line $y = x$, and the notation $f^{-1}(x)$				
Creating a sketch from information given or a context, including transferring a graph from screen to paper.				
Using technology to graph functions including their sums and differences.				
Using a calculator to sketch and locate key features of graphs of functions (max, min, zeros, intercepts, vertex, asymptotes, intersection of 2 curves)				
Modelling <ul style="list-style-type: none"> Linear $f(x) = mx + c$ Quadratics (axis of symmetry, vertex, zeros, x and y intercepts) $f(x) = ax^2 + bx + c$ Exponential growth and decay $f(x) = ka^x + c$, $f(x) = ka^{-x} + c$, $f(x) = ke^{rx} + c$ Including horizontal asymptotes Direct/inverse variation $f(x) = ax^n$ Cubic models $f(x) = ax^3 + bx^2 + cx + d$ Trig models $f(x) = a\sin(bx) + c$, $f(x) = a\cos(bx) + d$ 				
Modelling skills: <ul style="list-style-type: none"> Use the modelling process described above section to create, fit and use the theoretical models in section SL2.5 and their graphs. Develop and fit the model: <ul style="list-style-type: none"> Given a context recognize and choose an appropriate model and possible parameters. Determine a reasonable domain for a model. Find the parameters of a model. Comment on the appropriateness and reasonableness of a model. Justify the choice of a particular model, based on the shape of the data, properties of the curve and/or on the context of the situation. Reading, interpreting and making predictions based on the model. 				
Functions – composite and types of functions (one to one, many to one)				
Functions – finding inverses				
Transformations of graphs				
Translations: $y = f(x) + b$, $y = f(x - a)$				
Reflections: in the x axis $y = -f(x)$				
Reflections in the y axis $y = f(-x)$				
Vertical stretch with scale factor p : $y = pf(x)$.				
Horizontal stretch with scale factor $\frac{1}{a}$: $y = f(qx)$				
Composite transformations.				
Modelling				
Exponential models to calculate half-life.				
Natural logarithmic models $f(x) = a + b \ln x$				
Sinusoidal models $f(x) = a\sin(b(x - c)) + d$				

Logistic models $f(x) = \frac{L}{1+Ce^{-kx}}$, $L, C, k > 0$				
Scaling very large or small numbers using logarithms.				
Linearizing data using logarithms to determine if the data has an exponential or a power relationship using best-fit straight lines to determine parameters				
Interpretation of log-log and semi-log graphs				
Geometry and Trigonometry				
The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids.				
The size of an angle between two intersecting lines or between a line and a plane.				
Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles (SOHCAHTOA)				
Size of an angle between two intersecting lines or between a line and a plane				
Sine, cosine, and tangent ratios using special triangles				
Sine/cosine Rule (including the ambiguous case of sine rule)				
Area of a triangle				
Pythagoras				
Bearings				
Angles of elevation and depression				
Arc lengths and areas of sectors (not including radians)				
Equations of perpendicular bisectors				
Voronoi diagrams: sites, vertices, edges, cells. Addition of a site to an existing Voronoi diagram. Nearest neighbour interpolation				
Applications of the "toxic waste dump" problem				
Radians				
Arc lengths and areas of sectors (in radians)				
Definition of $\cos \theta$, $\sin \theta$ in terms of the unit circle				
Finding trig values of multiple angles of special angles using the unit circle				
Given the value of one trig function, find another (relationship between ratios)				
Identities $\sin^2 x + \cos^2 x = 1$ and $\tan x = \frac{\sin x}{\cos x}$				
Graphical methods of solving trigonometric equations in a finite interval.				
Geometric transformations of points in two dimensions using matrices: reflections, horizontal and vertical stretches, enlargements, translations and rotations				
Compositions of the above transformations				
Geometric interpretation of the determinant of a transformation matrix				
Concept of a vector and a scalar. Representation of vectors using directed line segments. Unit vectors; base vectors i, j, k .				
Components of a vector; column representation $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = v_1 i + v_2 j + v_3 k$				
Position vectors $\vec{OA} = a$				
Rescaling and normalizing vectors				
Vector equation of a line in two and three dimensions				
Vector applications to kinematics				
Modelling linear motion with constant velocity in two and three dimensions.				
Motion with variable velocity in two dimensions				
Definition and calculation of the scalar product of two vectors.				
The angle between two vectors; the acute angle between two lines.				
Definition and calculation of the vector product of two vectors.				
Geometric interpretation of $ \mathbf{v} \times \mathbf{w} $				
Components of vectors				
Graph theory: Graphs, vertices, edges, adjacent vertices, adjacent edges. Degree of a vertex				
Simple graphs; complete graphs; weighted graphs				
Directed graphs; in degree and out degree of a directed graph				
Subgraphs; trees				
Adjacency matrices				
Walks				
Number of k -length walks (or less than k -length walks) between two vertices				
Weighted adjacency tables				
Construction of the transition matrix for a strongly-connected, undirected or directed graph				
Tree and cycle algorithms with undirected graphs. Walks, trails, paths, circuits, cycles				
Eulerian trails and circuits				
Hamiltonian paths and cycles				
Minimum spanning tree (MST) graph algorithms				
Kruskal's and Prim's algorithms for finding minimum spanning trees				
Chinese postman problem and algorithm for solution, to determine the shortest route around a weighted graph with up to four odd vertices, going along each edge at least once				
Travelling salesman problem to determine the Hamiltonian cycle of least weight in a weighted complete graph				
Nearest neighbour algorithm for determining an upper bound for the travelling salesman problem				
Deleted vertex algorithm for determining a lower bound for the travelling salesman problem				
Statistics and Probability				
Concepts of population, sample, random sample, discrete and continuous data.				
Reliability of data sources and bias in sampling				
Interpretation of outliers				
Sampling techniques and their effectiveness				

Presentation of data (discrete and continuous): frequency distributions (tables)				
Cumulative frequency; cumulative frequency graphs; use to find median, quartiles, percentiles, range and interquartile range (IQR)				
Histograms				
Production and understanding of box and whisker diagrams				
Measures of central tendency (mean, median and mode)				
Estimation of mean from grouped data				
Modal class				
Measures of dispersion (interquartile range, standard deviation and variance).				
Effect of constant changes on the original data				
Quartiles of discrete data				
Scatter diagrams; lines of best fit, by eye, passing through the mean point				
Linear correlation of bivariate data				
Pearson's product-moment correlation coefficient and line of best fit				
Use of the equation of the regression line for prediction purposes (reliability)				
Equation of the regression line of y on x				
Interpret the meaning of the parameters, a and b , in a linear regression $y = ax + b$				
Basic probability and sample space				
Venn diagrams				
Tree diagram				
Two-way tables				
Addition formula				
Mutually exclusive events				
Independent events				
Conditional probability				
Concept of discrete random variables and their probability distributions				
Expected value (mean), for discrete data				
Applications such as fair game				
Binomial distribution (including mean and variance)				
Normal distribution (probability calculations and working backwards to find the value, mean or s.d.				
Spearman's rank correlation coefficient, r_s				
Awareness of the appropriateness and limitations of Pearson's product moment correlation coefficient and Spearman's rank correlation coefficient, and the effect of outliers on each				
Formulation of null and alternative hypotheses H_0 and H_1				
Significance levels				
p -values				
χ^2 test for independence, contingency tables, degrees of freedom, critical value				
χ^2 goodness of fit				
The t -test				
Use of the p -value to compare the means of two populations				
Using one-tailed and two-tailed tests				
Design of valid data collection methods, such as surveys and questionnaires				
Selecting relevant variables from many variables				
Choosing relevant and appropriate data to analyse				
Categorizing numerical data in a χ^2 table and justifying the choice of categorisation				
Choosing an appropriate number of degrees of freedom when estimating parameters from data when carrying out the χ^2 goodness of fit test				
Definition of reliability and validity. Reliability tests. Validity tests				
Non-linear regression				
Evaluation of least squares regression curves using technology				
Sum of square residuals (SS_{res}) as a measure of fit for a model				
The coefficient of determination R^2 . Evaluation of R^2 using technology				
Linear transformation of a single random variable				
Expected value of linear combinations of n random variables.				
Variance of linear combinations of n independent random variables.				
\bar{x} as an unbiased estimate of μ				
s_{n-1}^2 as an unbiased estimate of σ^2				
A linear combination of n independent normal random variables is normally distributed (sample)				
$X \sim N(\mu, \sigma^2) \Rightarrow \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$				
Central limit theorem				
Confidence intervals for the mean of a normal population				
Poisson distribution, its mean and variance				
Sum of two independent Poisson distributions has a Poisson distribution				
Critical values and critical regions				
Test for population mean for normal distribution				
Test for proportion using binomial distribution				
Test for population mean using Poisson distribution				
Use of technology to test the hypothesis that the population product moment correlation coefficient (ρ) is 0 for bivariate normal distributions.				
Type I and II errors including calculations of their probabilities				
Transition matrices and powers of transition matrices				
Regular Markov chains				
Initial state probability matrices				
Calculation of steady state and long-term probabilities by repeated multiplication of the transition matrix or by solving a system of linear equations.				

Calculus

Concept of a limit				
Derivative interpreted as gradient function and as rate of change.				
Increasing/Decreasing (including graphical representations of $f'(x) > 0, f'(x) < 0, f'(x) = 0$)				
$y = x^n$ differentiation technique (exponents are integers)				
Equations of Tangents and Normals				
Stationary maximum and minimum points.				
Optimisation problems in context				
Approximating areas using the trapezoidal rule.				
Composite functions differentiation techniques – chain rule ($(f(x))^n, \ln f(x), e^{f(x)}, \sin f(x), \cos f(x)$)				
Product and Quotient Rule				
Related rates of change				
Second derivative and using this to test for max/min				
Kinematics				
$\int x^n$ Integration technique				
Definite integrals				
Finding area under a curve (between the x axis) and between two curves				
Composite functions integration techniques ($(f(x))^n, e^{f(x)}, \sin f(x), \cos f(x)$, etc)				
Finding area under a curve (between the y axis)				
Integration by inspection/recognition/reverse chain rule				
Volume of revolution (between the x and y axis)				
Setting up a model/differential equation from a context.				
Solving by separation of variables				
Slope fields and their diagrams.				
Euler's method for finding the approximate solution to first order differential equations.				
Numerical solution of $\frac{dy}{dx} = f(x, y)$.				
Numerical solution of the coupled system $\frac{dx}{dt} = f_1(x, y, t), \frac{dy}{dt} = f_2(x, y, t)$				
Phase portrait for the solutions of coupled differential equations of the form: $\frac{dx}{dt} = ax + by$ $\frac{dy}{dt} = cx + dy$ Qualitative analysis of future paths for distinct, real, complex and imaginary eigenvalues. Sketching trajectories and using phase portraits to identify key features such as equilibrium points, stable populations and saddle points.				
Solutions of $\frac{d^2x}{dt^2} = f\left(x, \frac{dx}{dt}, t\right)$ by Euler's method.				